Test Centre : ______________________

Roll No. : ______________________

Name of the Candidate : ______________________

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Entrance Test for Ph.D. (Applied Mathematics) 2017

[PROGRAMME CODE : 50005]

Question Paper Series Code : A

QUESTION PAPER

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 30 questions of 1 mark each. All questions are compulsory. Part—B (objective-type) has 40 questions of 1 mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for rough work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Question Paper Series Code

   Write Question Paper Series Code **A** or **B** in the box and darken the appropriate circle.

   [ ] **A** or **B**

   ●

   Ⓑ

2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle. ●
4. Darken ONLY ONE CIRCLE for each question as shown below in the example :

***Example* :**

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

***Example* :**

ROLL NUMBER

1	3	5	7	2	0	2
●	①	①	①	①	①	①
②	②	②	②	●	②	●
③	●	③	③	③	③	③
④	④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●	⓪

PART—A

1. Every Cauchy sequence is convergent on

   a. real line

   b. complex plane

   c. both real line and complex plane

   d. None of the above

2. The value of $\lim_{h\to 0}\frac{1}{h}\int_{2}^{2+h}\sqrt{t^2+2}\,dt$ is

   a. 0

   b. $\sqrt{6}$

   c. $\sqrt{2}-2$

   d. $\sqrt{3}$

3. The value of

$$\lim_{h\to 0}\left\{\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right\}^{1/n}$$

   is

   a. e

   b. π

   c. $1/e$

   d. $1/\pi$

4. Let $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ be a function defined on $[-1, 1]$. Then which of the following statements is ***not*** true?

   a. f is a continuous function.

   b. f converges uniformly.

   c. f represents a bounded function.

   d. None of the above

5. Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$. Then the value of α for which $f(f(x)) = x$ is

a. $\sqrt{2}$

b. $-\sqrt{2}$

c. 1

d. -1

6. Let $f(x) = [x^2 - 3]$, where [.] denotes the greatest integer function. Then the number of points in the interval (1, 2), where the function is discontinuous, is

a. 4

b. 6

c. 2

d. ∞

7. Which one of the following d is ***not*** a metric on the set $\mathbb{R}$ of real numbers?

a. $d(x, y) = \frac{|x-y|}{1+|x-y|}$

b. $d(x, y) = \min(1, |x-y|)$

c. $d(x, y) = \max(1, |x-y|)$

d. $d(x, y) = \left|\frac{x}{1+|x|} - \frac{y}{1+|y|}\right|$

8. If $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

then

a. $\ker(L) = \{0\}$

b. $\ker(L) = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$

c. $\ker(L) = \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$

d. $\ker(L) = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$

9. If in ring $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} : a, b \in \mathbb{Z}\}$, then which of the following is correct?

a. 2 and $1+\sqrt{5}$ are irreducible and prime

b. 2 and $1+\sqrt{5}$ are irreducible but not prime

c. 2 and $1+\sqrt{5}$ are not irreducible but prime

d. 2 and $1+\sqrt{5}$ are neither irreducible nor prime

10. The rank of the matrix

$$\begin{pmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{pmatrix}$$

is

a. 1

b. 2

c. 3

d. 4

11. Let k be the sum of all the eigenvalues of a square matrix

$$\begin{pmatrix} -1 & 2 & -2 & 3 \\ 1 & 2 & 1 & 10 \\ -1 & -1 & 0 & 5 \\ 6 & -7 & -8 & 1 \end{pmatrix}$$

Then the value of k is

a. 2

b. 4

c. 6

d. 8

12. If in a group G, $a^3 = e$ and e is the identity of the group G and $aba^{-1} = b^2$ for a, b in G, then the order of b equals

a. 31

b. 23

c. 11

d. 7

13. In the ring $\mathbb{Z}$ of integers, let $A = \{0\}$. Then which of the following is incorrect?

a. A is a maximal ideal of $\mathbb{Z}$ but not a prime ideal of $\mathbb{Z}$

b. A is a prime ideal of $\mathbb{Z}$ but not a maximal ideal of $\mathbb{Z}$

c. A is neither maximal nor prime ideal of $\mathbb{Z}$

d. A is both maximal and prime ideal of $\mathbb{Z}$

14. Which of the following sets is ***not*** a vector sub-space of the vector space of 3×3 matrices over the field $\mathbb{R}$ of real numbers?

a. All upper triangular matrices of order 3

b. All symmetric matrices of order 3

c. All non-singular matrices of order 3

d. All matrices of order 3, the sum of whose diagonal elements is zero

15. The ordinary differential equation

$$\frac{dy}{dx} = \frac{2y}{x}$$

with the initial condition $y(0) = 0$ has

a. no solution

b. a unique solution

c. exactly two solutions

d. infinitely many solutions

16. For the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$$

the characteristic coordinates are given by

a. $\xi = x + 2t,\ \eta = x - 2t$

b. $\xi = x + 4t,\ \eta = x - 4t$

c. $\xi = x + 16t,\ \eta = x - 16t$

d. $\xi = x + 256t,\ \eta = x - 256t$

17. A homogenous linear differential equation with real constant coefficients, which has $y = xe^{-3x}\cos 2x + e^{-3x}\sin 2x$ as one of its solutions, is given by

a. $(D^2 - 6D + 13)\, y = 0$

b. $(D^2 + 6D + 13)\, y = 0$

c. $(D^2 - 6D + 13)^2\, y = 0$

d. $(D^2 + 6D + 13)^2\, y = 0$

18. The orthogonal trajectory to the family of circles $x^2 + y^2 = 2cx$ (c arbitrary) is described by the differential equation

a. $(y^2 - x^2)\, y' = xy$

b. $(y^2 - x^2)\, y' = 2xy$

c. $(x^2 - y^2)\, y' = 2xy$

d. $(x^2 + y^2)\, y' = 2xy$

19. The partial differential equation $y^3 u_{xx} - (x^2 + 1) u_{yy} = 0$ is

a. hyperbolic in $\{(x, y) : y > 0\}$

b. parabolic in $\{(x, y) : y < 0\}$

c. parabolic in $\{(x, y) : y > 0\}$

d. elliptic in $\mathbb{R}^2$

20. In the solution of the partial differential equation $(D^2 - 6DD' + 9D'^2)z = 6x + 2y$, the complementary function is given by

a. $\phi_1(y + 3x) + \phi_2(y - 3x)$

b. $\phi_1(3y + x) + \phi_2(3y - x)$

c. $\phi_1(y + 3x) + x\,\phi_2(y + 3x)$

d. $\phi_1(3y + x) + x\,\phi_2(3y + x)$

21. If $J_n(x)$ denotes the Bessel's function of first kind of order n, then which one of the following is true?

a. $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

b. $xJ_n'(x) = nJ_n(x) + xJ_{n+1}(x)$

c. $xJ_n'(x) = nJ_{n-1}(x) - xJ_{n+1}(x)$

d. $xJ_n'(x) = nJ_{n-1}(x) + xJ_{n+1}(x)$

22. The iterative formula to compute the reciprocal of a given positive real number α, using Newton-Raphson method is

a. $x_{n+1} = x_n(2 + \alpha x_n)$

b. $x_{n+1} = x_n^2(2 + \alpha x_n)$

c. $x_{n+1} = x_n^2(2 - \alpha x_n)$

d. $x_{n+1} = x_n(2 - \alpha x_n)$

23. The quadrature formula $\int_{-1}^{1} f(x)\,dx = f(\alpha) + \beta f(1)$ is exact for all polynomials of degree ≤ 1 for

a. $\alpha = 1, \beta = 1$

b. $\alpha = 1, \beta = -1$

c. $\alpha = -1, \beta = 1$

d. $\alpha = -1, \beta = -1$

24. The backward Euler method for solving the differential equation $y' = f(t, y)$ is

a. $y_{n+1} = y_n + hf(t_n, y_n)$

b. $y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$

c. $y_{n+1} = y_{n-1} + 2hf(t_n, y_n)$

d. $y_{n+1} = y_{n-1} + 2hf(t_n, y_{n+1})$

25. In geometric distribution, the relationship between the mean and variance is

a. variance < mean

b. 3 variance < 2 mean

c. variance = mean

d. variance > mean

26. If the density function of a random variable X is given by $f(x) = kx(2-x)$, $0 \leq x \leq 2$, then variance of the random variable is

a. 1/4

b. 1/5

c. 1/6

d. 1/7

27. A coin is tossed until a head appears. Expectation of the number of tosses is

a. 2

b. 6

c. 4

d. 8

28. A basic solution of a linear programming problem is called degenerate if

a. the value of at least one of the basic variables is non-zero

b. the value of all the basic variables is zero

c. the value of at least one of the basic variables is zero

d. the value of all the basic variables is non-zero

29. Which one of the following statements is ***not*** true about primal and dual?

a. The solution of the primal problem can be interpreted from the last simple table.

b. The decision variables in both primal and dual are positive.

c. If the primal problem is of maximization-type, then the dual will be of minimization-type.

d. The dual of dual is primal.

30. If the primal constraint is originally in equation form, then the corresponding dual variable is necessarily

a. non-negative

b. positive

c. negative

d. unrestricted

PART—B

31. If

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3 (3z+2)^2}$$

then which one of the following is true?

a. $z = 1$, $z = -2/3$ and $z = \infty$ are the poles of order 3, 2 and 3 respectively

b. $z = 1$, $z = -2/3$ are the poles of order 3 and 2 respectively

c. $z = \infty$, $z = 1$ and $z = -2/3$ are the poles of order 2, 3 and 2 respectively

d. $z = 1$, $z = -2/3$ and $z = \infty$ are the poles of order 3, 2 and 1 respectively

32. If $\mathbb{R}$ is a metric space with usual metric and $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$, then the boundary of A is the set

a. $\{0\}$

b. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$

c. $\{1, \infty\}$

d. $\{0, \infty\}$

33. Which one of the following is a compact subset of $\mathbb{R}$ but is ***not*** connected?

a. $(0, 1)$

b. $[1, 2)$

c. $[1, 2]$

d. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$

34. Let $d(x, y) = |x - y|$ be the metric defined on the set $\mathbb{N}$ of natural numbers. The open sphere $S_2(3)$ in $\mathbb{N}$ is the set

a. $\{1, 5\}$

b. $\{1, 10\}$

c. $\{2, 3, 4\}$

d. $\{1, 2, 3, 4, 5\}$

35. If $f_n(x) = \frac{x}{1+nx^2}$, and if $f(x) = 0$ for all real x, then which of the following is ***not*** true?

a. $< f_n'(x) >$ is point-wise convergent to some non-differentiable function

b. $f_n'(x) \to f'(x)$ for all x

c. $< f_n >$ converges point-wise to f

d. $< f_n >$ converges uniformly to f

36. The number of roots of $z^7 - 4z^3 + z + 1 = 0$ which lie interior to the unit circle $|z| = 1$ is

a. 3

b. 4

c. 5

d. 7

37. Which one of the following statements is incorrect?

a. Every subspace of T_0 topological space is T_0.

b. Every subspace of T_1 topological space is T_1.

c. Every subspace of normal topological space is normal.

d. Every subspace of Hausdorff topological space is Hausdorff.

38. Which one of the following statements is ***not*** true with reference to Lebesgue measurability?

a. Every continuous function is measurable.

b. Every characteristic function is measurable.

c. Every constant function is measurable.

d. Every monotonic function is measurable.

39. The value of the Lebesgue integral

$$\int_0^{\infty} \frac{dx}{\left(1+\frac{x}{n}\right)^n x^{\frac{1}{n}}}$$

as $n \to \infty$ equals

a. 1

b. ∞

c. e

d. 0

40. If $L : V \to W$ be a linear transformation with V and W finite dimensional, then which one of the following is correct?

a. If L is one-one, then $\dim(V) \geq \dim(W)$

b. If L is onto, then $\dim(V) \leq \dim(W)$

c. If L is onto, then $\dim(V) = \dim(W)$

d. If L is onto, then $\dim(V) \geq \dim(W)$

41. If $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $L(1, 1) = (0, 1)$, $L(-1, 1) = (2, 3)$, then the matrix of L with respect to standard basis of $\mathbb{R}^2$ is

a. $\begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

42. In a complex inner product space, which one of the following relations is always satisfied?

a. $(u, z_1 v_1 + z_2 v_2) = \bar{z}_1 (u, v_1) + \bar{z}_2 (u, v_2)$

b. $(u, z_1 v_1 + z_2 v_2) = z_1 (u, v_1) + \bar{z}_2 (u, v_2)$

c. $(u, z_1 v_1 + z_2 v_2) = \bar{z}_1 (u, v_1) + z_2 (u, v_2)$

d. $(u, z_1 v_1 + z_2 v_2) = z_1 (u, v_1) + z_2 (u, v_2)$

43. Which one of the following is ***not*** true?

a. The homomorphic image of an Abelian group is Abelian

b. The homomorphic image of a cyclic group is cyclic

c. The homomorphic image of a finite group is finite

d. The homomorphic image of an infinite group is infinite

44. If G be a finite group and simple, and if $O(G) = n$, then n can be which one of the following?

a. 33

b. 32

c. 31

d. 30

45. Let R be a ring with unity. Then R may ***not*** be commutative in which of the following situations?

a. When $(x+y)^2 = x^2 + y^2 + 2xy, \quad \forall x, y \in R$

b. When $x^2 = x, \quad \forall x \in R$

c. When $(xy)^2 = x^2y^2, \quad \forall x, y \in R$

d. When $2x = 0, \quad \forall x \in R$

46. Let V be the set of all 2×2 matrices over $\mathbb{R}$ and let $W_1 = \left\{\begin{pmatrix} x & z \\ 0 & y \end{pmatrix} : x, y, z \in \mathbb{R}\right\}$ and $W_2 = \left\{\begin{pmatrix} x & 0 \\ z & y \end{pmatrix} : x, y, z \in \mathbb{R}\right\}$. Then which of the following is ***not*** true?

a. $W_1 \cap W_2 = \{0\}$

b. V is a vector space over $\mathbb{R}$ and $\dim(V) = 2$

c. W_1 and W_2 are subspaces of V each of dimension 3

d. $V = W_1 + W_2$

47. Which one of the following sets of vectors is linearly dependent over $\mathbb{R}$?

a. $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$

b. $\{(2, -2, 5), (0, 8, -15), (3, 1, 0)\}$

c. $\{(0, 1, -2), (1, -1, 1), (1, 2, 1)\}$

d. $\{(1, -1, 1), (0, 1, 1), (1, 1, 1)\}$

48. The solution of $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ is

a. $(x + y + z)e^x = c$

b. $(x + y + z^2)e^x = c$

c. $(x + y + z^2)e^{x^2} = c$

d. $(x + y + z)e^{-x} = c$

49. Let n be a non-negative integer. The eigenvalues of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0$$

with boundary conditions $y(0) = y(\pi) = 0$ is

a. n

b. n^2

c. $n\pi$

d. $n^2\pi^2$

50. For the equation

$$x^3(x-2)\frac{d^2y}{dx^2} + x^3\frac{dy}{dx} + 6y = 0$$

which one of the following is true?

a. $x = 0$ is an ordinary point

b. $x = 2$ is an ordinary point

c. $x = 0$ is a regular singular point

d. $x = 2$ is a regular singular point

51. The region in which the partial differential equation

$$u_{xx} - \sqrt{y}u_{xy} + xu_{yy} = \cos(x^2 - 2y),\ y \geq 0$$

is hyperbolic is

a. $\{(x, y) : y > 4x\}$

b. $\mathbb{R}^2$

c. $\{(x, y) : y = 4x\}$

d. $\{(x, y) : y < 4x\}$

52. If $P_n(x)$ is the Legendre polynomial of degree n, then the value of $P_2(x)$ is

a. 1

b. x

c. $\frac{1}{2}(3x^2 - 1)$

d. $\frac{1}{2}(5x^3 - 3x)$

53. The solution of

$$\frac{(y-z)}{yz}\frac{\partial z}{\partial x} + \frac{(z-x)}{zx}\frac{\partial z}{\partial y} = \frac{x-y}{xy}$$

is

a. $x + y + z = f(xyz)$

b. $x^2 + y^2 + z^2 = f(xyz)$

c. $x - y + z = f(x^2 yz)$

d. $x^2 - y^2 + z^2 = f(xyz)$

54. The d'Alembert solution of the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ u(x, 0) = \sin x,\ u_t(x, 0) = 0$$

is

a. $u(x, t) = -\sin x \cos ct$

b. $u(x, t) = -\cos x \sin ct$

c. $u(x, t) = \sin x \cos ct$

d. $u(x, t) = \cos x \sin ct$

55. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is

a. $u(x, y) = f(x+y) + g(x-y)$

b. $u(x, y) = f(x+iy) + g(x-iy)$

c. $u(x, y) = e^{x}(f(x+iy) + g(x-iy))$

d. $u(x, y) = e^{-x}(f(x+iy) + g(x-iy))$

56. Let $g : [0, 1] \to \mathbb{R}$ be a three-times continuously differentiable function and the iterates defined by $x_{n+1} = g(x_n)$, $n \geq 0$ converge to the fixed point ξ of g. If the order of convergence is three, then

a. $g'(\xi) \neq 0$, $g''(\xi) \neq 0$

b. $g'(\xi) \neq 0$, $g''(\xi) = 0$

c. $g'(\xi) = 0$, $g''(\xi) \neq 0$

d. $g'(\xi) = 0$, $g''(\xi) = 0$

57. If $f : [0, 4] \to \mathbb{R}$ be a three-times continuously differentiable function, then the value of the divided difference $f[1, 2, 3, 4]$ is

a. $\frac{f''(\eta)}{3}$ for $\eta \in (0, 4)$

b. $\frac{f''(\eta)}{6}$ for $\eta \in (0, 4)$

c. $\frac{f'''(\eta)}{3}$ for $\eta \in (0, 4)$

d. $\frac{f'''(\eta)}{6}$ for $\eta \in (0, 4)$

58. A quadratic polynomial $f(x)$ is constructed by interpolating the data points (0, 1), (1, e) and (2, e^2). If $\sqrt{e}$ is approximated by using $f(x)$, then its approximate value is

a. $(3 - 6e - e^2)/8$

b. $(3 + 6e - e^2)/8$

c. $(3 + 6e - 2e^2)/8$

d. $(3 - 6e + 2e^2)/8$

59. For a sufficiently smooth function $f(x)$, a formula for estimating its derivative is given by

$$f'(x) = \frac{f(x+2h) - f(x-2h)}{4h} + \text{error term}$$

Which of the following expressions is correct for the error term?

a. $-f'(\xi)h$

b. $\frac{-f''(\xi)h^2}{2}$

c. $\frac{-2f'''(\xi)h^2}{3}$

d. $\frac{-f^{(4)}(\xi)h^4}{12}$

60. Using Euler's method and taking the step size as 0·5, the approximate solution corresponding to $x = 2$ for the initial value problem

$$\frac{dy}{dx} = 1 + \frac{y}{x}, \ y(1) = 1$$

is

a. 3·167

b. 2·048

c. 4·456

d. 2·218

61. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is six. The probability that it is actually a six is

a. 3/8

b. 1/5

c. 3/4

d. 5/6

62. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $(7^m + 7^n)$ is divisible by 5 is

a. 1/7

b. 1/8

c. 1/4

d. 1/49

63. If (X, Y) is uniformly distributed over the semi-circle bounded by $y = \sqrt{1 - x^2}$ and $y = 0$, then the value of $E(X/Y)$ is

a. 1

b. 0

c. $\frac{4}{3\pi}$

d. $\frac{2}{3\pi}$

64. Suppose a random variable X has a binomial distribution $B(6, \frac{1}{2})$. The most likely outcome for X is

a. 6

b. 4

c. 5

d. 3

65. If

$$f(x) = \begin{cases} 0 & , \quad x \le -1 \\ m(x+1) & , \quad -1 < x \le 3 \\ 4m & , \quad 3 < x \le 4 \\ 0 & , \quad x > 4 \end{cases}$$

represents the density function, then the value of x, when the mean deviation of this distribution is the least, is

a. $\sqrt{3}+1$

b. $\sqrt{3}-1$

c. $2\sqrt{3}-1$

d. $2\sqrt{3}+1$

66. Consider the primal problem

Maximize $Z = 4x + 3y$

subject to

$$x + y \le 8$$
$$2x + y \le 10$$
$$x \ge 0$$
$$y \ge 10$$

together with its dual. Then which one of the following statements is correct?

a. Primal and dual both are infeasible

b. Primal and dual both are feasible

c. Primal is feasible but dual is infeasible

d. Primal is infeasible but dual is feasible

67. If $S_1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\}$ and $S_2 = \{(x, y) \in \mathbb{R}^2 : y \le x^2\}$, then which one of the following statements is correct?

a. S_1 is a convex set but S_2 is not a convex set

b. S_2 is a convex set but S_1 is not a convex set

c. Neither S_1 nor S_2 is a convex set

d. S_1 and S_2 are both convex sets

68. If a primal linear programming problem admits an optimal solution, then the corresponding dual problem

a. does not have a feasible solution

b. has a feasible solution but does not have any optimal solution

c. does not have a convex feasible region

d. has an optimal solution

69. For a linear programming problem

Minimize $Z = x - y$

subject to

$$2x + 3y \le 6$$
$$0 \le x \le 3$$
$$0 \le y \le 3$$

which of the following represents the correct number of extreme points of its feasible region and of basic feasible solutions respectively?

a. 3 and 3

b. 4 and 4

c. 3 and 5

d. 4 and 5

70. Consider the linear programming problem

Minimize $Z = -2x - 5y$

subject to

$$3x + 4y \le 5$$
$$x \ge 0$$
$$y \ge 0$$

Then which one of the following statements is correct?

a. Set of feasible solutions is empty.

b. Set of feasible solutions is non-empty but there is no optimal solution.

c. Optimal value is attained at (0, 5/4).

d. Optimal value is attained at (5/3, 0).

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