14

QUESTION PAPER SERIES CODE
B

Centre Name : ____________________

Roll No. : ____________________

Name of Candidate : ____________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2015

[PROGRAMME CODE : PAM]

Time : 3 hours | *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Centre Name in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(v) **One-fourth (1/4) of marks assigned to any question in Part—A and Part—B will be deducted for wrong answers.**

(vi) Symbols have their usual meanings.

(vii) **Please darken the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR/Answer Sheet in the space provided.**

(viii) Part—A and Part—B (Multiple Choice) questions should be answered on the OMR/Answer Sheet.

(ix) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(x) Calculators and Log Tables may be used. Mobile Phones are **NOT** allowed.

(xi) Pages at the end have been provided for Rough Work.

(xii) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xiii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B and darken appropriate circle.

   [ ] A or B

   Ⓐ

   ●

   Programme Code

   Write Programme Code out of 14 codes given and darken appropriate circle.

   Write Programme Code [ ]

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **One-fourth (1/4) of marks assigned to any question will be deducted for wrong answers in multiple choice questions.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

    ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. The general solution to the PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$, is

(a) $u(x, y) = f(y+x) + g(2y+x)$

(b) $u(x, y) = f(y+3x) + g(y-3x)$

(c) $u(x, y) = f(y+x) + g(y-x) + F(y+ix) + G(y-ix)$

(d) $u(x, y) = f(y-x) + xg(y-x)$

2. If $U_k(x, y)$, $k = 1(1)n$ are solutions of the homogeneous linear partial differential equations $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$, then which of the following is also a solution?

(a) $\sum_{k=1}^{n} a_k U_k$

(b) $\sum_{k=1}^{n} a_k U_k^2$

(c) $\sum_{k=1}^{n} a_k U_k^3$

(d) $\sum_{k=1}^{n} a_k U_k^4$

3. The one-dimensional diffusion equation $\frac{\partial^2 U}{\partial x^2} = \frac{1}{k}\frac{\partial U}{\partial t}$, where $U(x, t) \to 0$, as $t \to \infty$, admits the solution

(a) $u(x, t) = \sum_{n=0}^{\infty} C_n \sin(nx + \varepsilon_n) e^{-n^2 kt}$

(b) $u(x, t) = \sum_{n=0}^{\infty} C_n \cos(nx + \varepsilon_n) e^{-n^2 kt}$

(c) $u(x, t) = \sum_{n=0}^{\infty} C_n \exp(nx + \varepsilon_n) e^{-n^2 kt}$

(d) $u(x, t) = \sum_{n=0}^{\infty} C_n \cosh(nx + \varepsilon_n) e^{-n^2 kt}$

4. Let y be the solution of the initial value problem $\frac{d^2 y}{dx^2} + y = \cos(2x)$, $y(0) = 1$, $y'(0) = -2$. Let the Laplace transform of y be $F(s)$. Then the value of $F(1)$ is

(a) 10/3

(b) 2/5

(c) −2/5

(d) 3/10

5. Which of the following represents two-dimensional Laplacian operator in polar coordinate?

(a) $\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}$

(b) $\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$

(c) $\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$

(d) $\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$

6. The numerical value obtained by applying the seven-point trapezoidal rule to the integral $\int_0^6 \frac{1}{1+x^2}\, dx$ is

(a) 1·4108

(b) 3·4108

(c) 6·4108

(d) 8·4108

7. Using the Jacobi iteration method with the initial guess $\{x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0\}$, the second approximation $\{x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\}$ for the solution of the system of equations

$$
\begin{aligned}
20x_1 + x_2 - 2x_3 &= 17 \\
3x_1 + 20x_2 - x_3 &= -18 \\
2x_1 - 3x_2 + 20x_3 &= 25
\end{aligned}
$$

is

(a) $x_1^{(2)} = 9 \cdot 02$, $x_2^{(2)} = -0 \cdot 965$, $x_3^{(2)} = 1 \cdot 1515$

(b) $x_1^{(2)} = 1 \cdot 02$, $x_2^{(2)} = -0 \cdot 965$, $x_3^{(2)} = 1 \cdot 1515$

(c) $x_1^{(2)} = 1 \cdot 02$, $x_2^{(2)} = 0 \cdot 965$, $x_3^{(2)} = 1 \cdot 1515$

(d) $x_1^{(2)} = 1 \cdot 02$, $x_2^{(2)} = -0 \cdot 965$, $x_3^{(2)} = 4 \cdot 1515$

8. The fourth-order Runge-Kutta method given by

$$y_{k+1} = y_k + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

is used to solve the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 1$. If $y(0 \cdot 2) = 1 \cdot 2428$ is obtained by taking the step size $h = 0 \cdot 2$, then the value of K_2 is

(a) 8·41

(b) 3·41

(c) 5·24

(d) 0·24

9. Using Euler's method taking the step size as 0·1, the approximate value of y obtained corresponding to $x = 0 \cdot 2$ for the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 1$, is

(a) 0·85

(b) 5·89

(c) 3·87

(d) 1·36

10. Three groups of children contain respectively

3 girls, 1 boy; 2 girls, 2 boys; 1girl, 3 boys

One child is selected at random from each group. The chance that the three selected consist of 1 girl and 2 boys is

(a) 1/4

(b) 9/32

(c) 11/32

(d) 13/32

11. If a random variable X has a Poisson distribution such that $p(1) = p(2)$, then the mean of the distribution is

(a) 2

(b) 3/2

(c) 1

(d) 1/2

12. Addition of a new constraint to an LPP can affect

(a) feasibility condition only

(b) optimality condition only

(c) feasibility and optimality conditions both

(d) neither feasibility nor optimality condition

13. Let in simplex method for an LPP, the variable x_j leave the basis in some iteration. Then

(a) x_j can enter the basis in the next iteration

(b) x_j cannot enter the basis in the next iteration

(c) Both the above cases are possible depending upon the LPP

(d) None of the above

14. Let A be the set of all rational numbers with discrete metric d. Then which of the following is not true in the metric space (A, d)?

(a) Every subset of A is open

(b) Every subset of A is closed

(c) (A, d) is incomplete

(d) (A, d) is disconnected

15. Which one of the following is not true?

(a) The sum of two Lebesgue measurable functions is measurable

(b) The product of two Lebesgue measurable functions is measurable

(c) The composite of two Lebesgue measurable functions is measurable

(d) Maximum of two Lebesgue measurable functions is measurable.

16. If X^{**} denotes the second dual space of a normed linear space X, then which of the following is not true?

(a) $(\mathbb{C}^n)^{**} = \mathbb{C}^n$

(b) $(l_1)^{**} = l_1$

(c) $(l_2)^{**} = l_2$

(d) $(l_3)^{**} = l_3$

(The notations stand their usual meanings.)

17. Let f be defined on the set $\mathbb{C}$ of complex numbers as

$$f(z) = \begin{cases} \dfrac{x^3 y(y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Then f does not satisfy which of the following?

(a) f is analytic at $z = 0$

(b) f is continuous at $z = 0$

(c) Cauchy-Riemann equations are satisfied at $z = 0$

(d) $\dfrac{f(z) - f(0)}{z} \to 0$ as $z \to 0$ along x-axis or y-axis

18. The value of the integral

$$\frac{1}{2\pi i}\int_C \frac{8z}{(z-1)(z+3)(z-5)^2}\,dz, \text{ where } C = \{z : |z| = 2\}$$

equals

(a) 1/8

(b) 1/4

(c) 1/2

(d) 1/32

19. If $< a_n >$ is a sequence of real numbers defined as

$$a_1 = 1$$
$$a_{n+1} = \sqrt{6 + a_n},\ n = 1, 2, \ldots$$

then

(a) $< a_n >$ converges to 2

(b) $< a_n >$ converges to 3

(c) $< a_n >$ converges to 6

(d) $< a_n >$ is oscillatory and not convergent

20. For a similar matrix A and B

(a) A, B have same characteristic polynomial

(b) A, B may have different characteristic polynomials

(c) A, B have same eigenvalue

(d) A, B have different eigenvalues

21. The homomorphisms from Z_{20} onto Z_{10} are

(a) 4

(b) 20

(c) 8

(d) 10

22. The alternating group A_4 on 4 symbols has a normal subgroup of order

(a) 2

(b) 3

(c) 4

(d) 6

23. The incorrect statement for a non-Abelian group G of order p^3, where p is prime, is

(a) $Z(G)$; the center of the group G has p elements

(b) G'; the commutator subgroup of G has p elements

(c) $Z(G) = G'$

(d) None of the above

24. The elements of order 5 in S_7 are

(a) 120

(b) 21

(c) 504

(d) 24

25. The order of 2-Sylow subgroup A_4 is

(a) 4

(b) 6

(c) 2

(d) None of the above

26. If G is a group of order 91, then which statement is false?

(a) G has one subgroup of order 7

(b) G has two subgroups of order 13

(c) G has subgroups of order 7 and 13

(d) None of the above

27. The $n \times n$ matrix P is idempotent if $P^2 = P$ and orthogonal if $P'P = I$. Which of the following is false?

(a) If P and Q are orthogonal $n \times n$ matrices, then PQ is orthogonal

(b) If P and Q are idempotent $n \times n$ matrices and $PQ = QP = O$, then $P + Q$ is idempotent

(c) If P is idempotent, then $-P$ is idempotent

(d) $P = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ is orthogonal

28. Which of the following is a parabolic partial differential equation?

(a) $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial x}$

(b) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

(c) $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} - \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial x}$

(d) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = 0$

29. Let $U(x, t)$ be a function defined for $a \le x \le b$, $t > 0$. Then which of the following is Laplace transformation of $\dfrac{\partial U}{\partial t}$?

(a) $sL\{U(x, t)\} - U(x, 0)$

(b) $sL\{U(x, t)\} - U(x, \infty)$

(c) $sL\{U(x, t)\}$

(d) $sL\{U(x, t)\} + U(x, 0)$

30. The solution to the linear PDE $\dfrac{\partial^4 u}{\partial x^4} - \dfrac{\partial^4 u}{\partial y^4} = 0$, is

(a) $u(x, y) = f(y + x) + g(y + ix)$

(b) $u(x, y) = f(y + 3x) + g(y - 3x)$

(c) $u(x, y) = f(y + x) + g(y - x) + F(y + ix) + G(y - ix)$

(d) $u(x, y) = f(y + x) + g(y + 2ix)$

PART—B

31. The differential equation of the family of trajectories of the family of curve given by $F\left(x, y, \frac{dy}{dx}\right) = 0$, is

(a) $F\left(x, y, -\frac{dy}{dx}\right) = 0$

(b) $F\left(x, y, \frac{dx}{dy}\right) = 0$

(c) $F\left(x, y, \frac{dy}{dx}\right) = 0$

(d) $F\left(x, y, -\frac{dx}{dy}\right) = 0$

32. Which of the following satisfies the initial value problem $\frac{dy}{dx} = (4x + y + 1)^2$, $y(0) = 1$?

(a) $4x - y + 1 = 2\tan(2x + \pi/4)$

(b) $4x + y + 1 = \tan(x + \pi/4)$

(c) $x - y + 1 = 2\tan(2x + \pi/4)$

(d) $4x + y + 1 = 2\tan(2x + \pi/4)$

33. The characteristic of the first-order equation $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x$ is given by the curve

(a) $y = ce^{-x}$

(b) $y = ce^{x}$

(c) $y = \sin(x)$

(d) $y = \cosh(x)$

34. The solution of the equation $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$ is

(a) $x^2 + 2y^2 + z^2 = f(lx + 5my + nz)$

(b) $x^2 + y^2 z^2 = f(lx + my + nz)$

(c) $x^2 - y^2 - z^2 = f(lx + my + nz)$

(d) $x^2 + y^2 + z^2 = f(lx + my + nz)$

35. Which of the following defines the central difference operators?

(a) $\delta y(x) = y\left(x+\frac{h}{2}\right)y\left(x+\frac{3h}{2}\right)$

(b) $\delta y(x) = y\left(x+\frac{h}{2}\right)y\left(x-\frac{h}{2}\right)$

(c) $\delta y(x) = y\left(x+\frac{h}{2}\right)-y\left(x-\frac{h}{2}\right)$

(d) $\delta y(x) = y\left(x+\frac{h}{2}\right)+y\left(x-\frac{h}{2}\right)$

36. The Newton's scheme of iteration for finding the square root of a positive integer N is

(a) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$

(b) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$

(c) $x_{n+1} = \frac{1}{2}\left(x_{n+1} + \frac{N}{x_n}\right)$

(d) $x_{n+1} = \frac{1}{2}\left(x_{n-1} + \frac{N}{x_n}\right)$

37. A relation between the differences of an unknown function at one or more general values of the argument is known as

(a) partial differential equation

(b) linear integral equation

(c) differential equation

(d) difference equation

38. The Runge-Kutta method of second-order is same as

(a) Euler method

(b) modified Euler method

(c) Taylor method

(d) fourth-order Runge-Kutta method

39. Which of the following is true for the solution of $AX = b$, where

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} ?$$

(a) Jacobi method converges

(b) Jacobi method diverges

(c) Gauss-Seidel method converges

(d) Both Jacobi method and Gauss-Seidel method converge

40. The contents of urns I, II and III are as follows :

Urn I : 1 white, 2 black and 3 red balls

Urn II : 2 white, 1 black and 1 red balls

Urn III : 4 white, 5 black and 3 red balls

One urn is chosen at random and two balls drawn. They happen to be white and red. The probability that they come from urn III is

(a) 55/118

(b) 33/118

(c) 15/59

(d) 13/59

41. Suppose X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx, & 0 \le x < 2 \\ 2k, & 2 \le x < 4 \\ -kx + 6k, & 4 \le x < 6 \end{cases}$$

Then the mean value of X is

(a) 4

(b) 3

(c) 2

(d) 1

42. Let

$$f(x, y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0, \; y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

be the probability distribution function of the random variables X and Y. Then which one of the following is true?

(a) $E(XY)$ does not exist

(b) $E(Y/X)$ does not exist

(c) $E(Y)$ does not exist

(d) All the three $E(XY)$, $E(Y/X)$, $E(Y)$ exist

43. Let $P(Y = \pm 1) = \frac{1}{2}$ and define the sequence $< X_n >$ by

$$X_n = \begin{cases} Y, & \text{if } n \text{ is odd} \\ -Y, & \text{if } n \text{ is even} \end{cases}$$

Then the sequence $< X_n >$

(a) converges in distribution and not in probability

(b) converges in probability but not in distribution

(c) neither converges in probability nor in distribution

(d) converges both in probability and distribution

44. Consider the following LPP :

Maximize $Z = x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 2$$
$$-x_1 + x_2 \leq 4$$

x_1 is unrestricted and $x_2 \geq 0$.

Then which of the following is the best basic feasible objective function value?

(a) 8

(b) 11

(c) 6

(d) None of the above

45. The optimal value of the objective function for the following LPP

Minimize $Z = 10x_1 + 4x_2 + 5x_3$

subject to

$$5x_1 - 7x_2 + 3x_3 \geq 50$$

$$x_1, x_2, x_3 \geq 0$$

is

(a) 50

(b) 250/3

(c) 270/4

(d) None of the above

46. Let the optimal basic solution to the primal be degenerate. Then the dual problem has

(a) alternative optimal solutions

(b) no solution

(c) an unbounded solution

(d) None of the above

47. In a maximization LPP, the variable corresponding to minimum ratio with solution column leaves the basis. This ensures

(a) largest rise in the objective function

(b) that the next solution will be a BFS

(c) that the next solution will not be unbounded

(d) None of the above

48. In two-phase simplex method, original problem may be maximization or minimization problem but the phase–I problem

(a) is always a minimization problem

(b) is always a maximization problem

(c) may be minimization or maximization problem

(d) None of the above

49. Let $\mathbb{R}$ be the set of real numbers and let $\Im$ be the usual topology on $\mathbb{R}$. Let f be a function defined from $(\mathbb{R}, \Im) \to (\mathbb{R}, \Im)$ as $f(x) = 1$ for all x. Then f satisfies which of the following?

(a) f is continuous and closed but not open

(b) f is continuous and open but not closed

(c) f is open and closed but not continuous

(d) f is continuous, closed and open

50. Which of the following is not true in the context of topological spaces?

(a) Continuous image of a compact set is compact

(b) Continuous image of a bounded set is bounded

(c) Continuous image of a connected set is connected

(d) Continuous image of a sequentially compact set is sequentially compact

51. Let $< f_n >$ be a sequence of functions defined on the set $\mathbb{R}$ of real numbers as :

$$f_n(x) = \begin{cases} nx(1-x^2)^n, & 0 < x < 1, \quad n = 1, 2, 3, \ldots \\ 0, & \text{otherwise}, \quad n = 1, 2, 3, \ldots \end{cases}$$

Then which of the following is not true?

(a) Each f_n is Lebesgue integrable

(b) $f_n(x) \to f(x)$ for all x and f is bounded measurable

(c) f is Lebesgue integrable

(d) $\lim \int f_n = \int f$

52. Let $H = C[0, 1]$ be the space of all complex-valued continuous functions with mapping

$$H \times H \to \mathbb{C} \text{ as } (f, g) = \int_0^1 f(x)\overline{g(x)}\,dx$$

Then which of the following is not true?

(a) H is an inner product space

(b) H is a normed linear space with $\|f\| = (\int |f|^2)^{1/2}$

(c) H is complete

(d) The norm defined in (b) satisfies parallelogram law

53. Suppose f is an entire function and $|f(z)| < a + b|z|^{1/2}$ for all z. Then which one of the following is true?

(a) f is a constant function

(b) f can be a polynomial of degree 1

(c) f can be a polynomial of degree 2

(d) f can be a polynomial of degree 3

54. If

$$f(z) = \begin{cases} \dfrac{1-\cos z}{z^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

then which one of the following is true?

(a) $z = 0$ is a pole of order 1

(b) $z = 0$ is a pole of order 2

(c) $z = 0$ is a removable singularity

(d) $z = 0$ is an essential singularity

55. Let

$$f(x) = \begin{cases} \dfrac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \leq x < 0 \\ \dfrac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$$

If $f(x)$ is continuous in the interval $[-1, 1]$, then p equals

(a) 1/2

(b) −1/2

(c) 1

(d) −1

56. Let

$$f(x) = \begin{cases} x^3 \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then f satisfies which of the following?

(a) f is differentiable nowhere

(b) f is differentiable everywhere except at $x = 0$

(c) f is differentiable once at $x = 0$ but not twice differentiable at $x = 0$

(d) f is twice differentiable at $x = 0$ but not thrice differentiable at $x = 0$

57. A real quadratic form $X^T AX$ is positive semidefinite, if

(a) all eigenvalues of $A \geq 0$

(b) all eigenvalues of $A \leq 0$

(c) all eigenvalues of $A = 0$

(d) None of the above

58. The following vectors

$$(1/4, 0, -1/4), (1/3, -1/3, 0) \text{ and } (0, 1/2, -1/2)$$

are

(a) linearly independent

(b) linearly dependent

(c) constants

(d) None of the above

59. Let T be a linear operator on a finite dimensional vector space V and c is any scalar. Then c is a characteristic value of T, if

(a) the operator $(T - cI)$ is singular

(b) the operator $(T - cI)$ is non-singular

(c) the operator $(T - cI)$ is identity

(d) the operator $(T - cI)$ is zero

60. If n is the order of element a of group G, then $a^m = e$ (an identity element) if and only if

(a) m/n

(b) n/m

(c) $n = m$

(d) n does not divide m

61. Consider the following statements :

Statement A : All cyclic groups are Abelian.

Statement B : The order of a cyclic group is same as the order of its generator.

Choose the correct option.

(a) A and B are false

(b) A is true, B is false

(c) B is true, A is false

(d) A and B are true

62. Consider the following statements :

Statement A : Every isomorphic image of a cyclic group is cyclic.

Statement B : Every quotient group of a cyclic group is cyclic.

Choose the correct option.

(a) Both A and B are false

(b) Only A is true

(c) Only B is true

(d) Both A and B are true

63. For a group G of order 15, the number of 3-Sylow subgroups of G is

(a) 0

(b) 1

(c) 3

(d) 5

64. Let R is a commutative ring with unity and only ideals are (0) and R. Then

(a) R is finite integral domain

(b) R is integral domain

(c) R is division ring

(d) R is a field

65. The homomorphism f from ring R onto ring R' is an isomorphism if and only if kernel of f is

(a) $\{0\}$

(b) R

(c) R'

(d) None of the above

66. Which of the following statements is false?

(a) $F[x]$ is an integral domain

(b) $F[x]$ is a Euclidean ring

(c) $F[x]$ is a principal ideal ring

(d) $F[x]$ is not a group

67. The solution of the linear ordinary differential equation $y'' - 3y' + 2y = e^x$ is

(a) $a_1 e^x + a_2 e^{2x} - xe^x$

(b) $a_1 e^x + a_2 e^{-2x} - xe^x$

(c) $a_1 e^{-x} + a_2 e^{2x} - xe^x$

(d) $a_1 e^x + a_2 e^{2x} + xe^x$

68. The solution of the initial value problem $\frac{dy}{dx} = e^{x+y}$, $y(1) = 1$ at $x = -1$ is

(a) 2

(b) 0

(c) −1

(d) 1

69. The general solution of the differential equation $2yzdx + zxdy - xy(1+z)dz = 0$ is

(a) $x^2 y = cze^z$

(b) $x^2 y^2 = cze^z$

(c) $xy = ce^z$

(d) $x^2 y = cz$

70. Which of the following is true?

(a) All eigenvalues of Sturm-Liouville problem are zero

(b) Eigenvalues of Sturm-Liouville problem are imaginary

(c) All eigenvalues of Sturm-Liouville problem are real

(d) All eigenvalues of Sturm-Liouville problem are unity

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SPACE FOR ROUGH WORK

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