7

QUESTION PAPER SERIES CODE
B

Centre Name : ______________________

Roll No. : ______________________

Name of Candidate : ______________________

S A U

Entrance Test for M.Sc. (Applied Mathematics), 2015

[PROGRAMME CODE : MAM]

Time : 3 hours *Maximum Marks* : 100

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Centre Name in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 60 questions of **1** mark each. All questions are compulsory.

(v) **One fourth of marks assigned to any question in Part A and Part B will be deducted for wrong answers.**

(vi) Symbols have their usual meanings.

(vii) **Please darken the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR in the space provided.**

(viii) Part—A and Part—B (Multiple Choice) questions should be answered on OMR Sheet.

(ix) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(x) Calculators and Log Tables may be used. Mobile Phones are **NOT allowed**.

(xi) Pages at the end have been provided for Rough Work.

(xii) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xiii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B and darken the appropriate circle.

   [ ] A or B

   Ⓐ

   ●

   Programme Code

   Write Programme Code out of 14 codes given and darken the appropriate circle.

   Write Programme Code [ ]

MEC	○	MAM	●	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	○
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **One fourth of marks assigned to any question will be deducted for wrong answers.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

1. The solution of the differential equation $(x\sin(y/x))\,dx - (y\sin(y/x) - x)\,dy = 0$ is

(a) $\cos(y/x) = 0$

(b) $\sin(y/x) = 0$

(c) $\cos(y/x) - \log x =$ constant

(d) $\sin(y/x) - \log x =$ constant

2. The solution of the given partial differential equation $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial x^4} = 0$ is

(a) $f_1(y+x) + f_2(y-x) + f_3(y+ix) + f_4(y-ix)$

(b) $f_1(y+x) + f_2(y-x)$

(c) $f_1(y+ix) + f_2(y-ix)$

(d) None of the above

3. The differential equation $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16\log(x+2y)$ has the solution

(a) $z = f_1(2y+x) + xf_2(2y+x) + 2x^2\log(x+2y)$

(b) $z = 2x^2\log(x+2y)$

(c) $z = 2f_1(2y+x) + 2x^2\log(x+2y)$

(d) None of the above

4. The solution of $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) + z = 0$ is

(a) $z = 2e^{-x}f_1(y+x)$

(b) $z = e^{-x}f_1(y-x) + xe^{-x}f_2(y-x)$

(c) $z = f_1(y-x) + x\,f_1(y-x)$

(d) None of the above

5. The equation $\frac{\partial z}{\partial x} e^y = \frac{\partial z}{\partial y} e^x$ gives the general solution

(a) $z = ae^x + be^x$

(b) $z = e^x + e^y$

(c) $z = a(e^x + e^y) + b$

(d) None of the above

6. Solution of $p^2 + px + py + xy = 0,\ p = \frac{\partial z}{\partial x}$ is

(a) $(2y + x^2 + c_1)(x + \log y) = 0$

(b) $(2y + x^2 - c_1)(x + \log y - c_2) = 0$

(c) $(2y + x^2 - c_1) = 0$

(d) None of the above

7. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 1$ has the solution

(a) $z = f_1(y) + e^{-x} f_2(y + x) + x$

(b) $z = f_1(y) + e^{-x} f_2(y + x)$

(c) $z = x$

(d) None of the above

8. If x_0 be the initial approximation to the root of the equation $f(x) = 0$, then the next approximation x_1 to the root by Newton-Raphson method is

(a) $x_0 + \frac{f(x_0)}{f'(x_0)}$

(b) $x_0 - \frac{f(x_0)}{f'(x_0)}$

(c) $x_0 - \frac{f(x_1)}{f'(x_0)}$

(d) $x_1 - \frac{f(x_0)}{f'(x_0)}$

9. The order of convergence to bisection method is

(a) quadratic

(b) cubic

(c) linear

(d) quintic

10. The bounds of error in quadratic interpolation of equispaced data $\{x_k, f(x_k)\}$, $k = 0$, 1 and 2, is given by

(a) $\frac{h^3}{9\sqrt{3}} \max|f'(x)|$, $h = x_2 - x_1 = x_1 - x_0$

(b) $\frac{h^3}{9\sqrt{3}} \max|f''(x)|$, $h = x_2 - x_1 = x_1 - x_0$

(c) $\frac{h^3}{\sqrt{3}} \max|f'''(x)|$, $h = x_2 - x_1 = x_1 - x_0$

(d) $\frac{h^3}{9\sqrt{3}} \max|f'''(x)|$, $h = x_2 - x_1 = x_1 - x_0$

11. Which of the following relations is true if Δ denotes the forward difference operator?

(a) $\Delta\left(\frac{1}{g_k}\right) = -\frac{\Delta g_k}{g_k g_{k+1}}$

(b) $\Delta\left(\frac{1}{g_k}\right) = \frac{\Delta g_k}{g_k g_{k+1}}$

(c) $\Delta\left(\frac{1}{g_k}\right) = -\frac{\Delta g_k}{g_k}$

(d) $\Delta\left(\frac{1}{g_k}\right) = -\frac{\Delta g_k}{g_{k+1}}$

12. Euler method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(0) = y_0$ is given by the equation

(a) $y_{n+1} = y_n + hf(x_n, y_n)$

(b) $y_{n+1} = y_n - hf(x_n, y_n)$

(c) $y_{n+1} = y_n + \frac{h}{2} f(x_n, y_n)$

(d) $y_{n+1} = y_n + 2hf(x_n, y_n)$

13. The average daily wage of 50 workers of a factory was ₹ 50 with standard deviation of ₹ 40. Each worker is given a rise of ₹ 20. Then their new average daily wage (X) and standard deviation (σ) are

(a) $X =$ ₹ 220 and $\sigma = 1$

(b) $X =$ ₹ 240 and $\sigma = \sqrt{2}$

(c) $X =$ ₹ 220 and $\sigma = 40$

(d) $X =$ ₹ 220 and $\sigma = \sqrt{40}$

14. Apples are sold at the rates of ₹ 80, ₹ 100, ₹ 120, and ₹ 150 per kilogram in four different months. Assuming that equal amounts are spent on apples by a family in the four months, the average price in rupees/month is

(a) 112·50

(b) 106·70

(c) 110·00

(d) None of the above

15. Let C be an event which is neither a certainty nor impossibility. Its probability is such that $P(C) = 1 + \lambda + \lambda^2$ and $P(C') = (1 + \lambda)^2$ in terms of an unknown parameter λ, where C' is the complement of C. What is the value of $P(C)$?

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

16. A constraint that does not affect the feasible region is a

(a) non-negativity constraint

(b) redundant constraint

(c) standard constraint

(d) slack constraint

17. If there is no feasible region of an LPP, then we say that the problem has

(a) infinite solutions

(b) no solution

(c) unbounded solution

(d) None of the above

18. In canonical form of an LPP, the availability vector b

(a) is restricted to > 0

(b) is restricted to < 0

(c) is equal to 0

(d) No restriction of > 0, < 0 or $= 0$ is required

19. If the primal constraint is originally in equation form, the corresponding dual variable is necessarily

(a) non-negative

(b) positive

(c) unrestricted

(d) None of the above

20. For an LPP, if one or more basic variable(s) is/are zero, then the basic feasible solution is called

(a) non-degenerate

(b) degenerate

(c) unbounded

(d) None of the above

21. Given that composition of two functions $f \circ g$ is a continuous function. Then

(a) both f and g are necessarily continuous

(b) f is necessarily continuous

(c) g is necessarily continuous

(d) both f and g could be discontinuous

22. Which of the following functions is differentiable in the interval (1, 3)?

(a) $f(x) = |x-1| + |x-3|$

(b) $f(x) = |x-2|$

(c) $f(x) = \dfrac{1}{x^2 - 4x + 4}$

(d) $f(x) = \log(x-2)$

23. The function $w = (t-15)^2$ denotes the amount of water in a tank t minutes after it started to drain. The average rate at which the water flows out during the first 5 minutes is

(a) 5000 units

(b) 4500 units

(c) 2500 units

(d) 2000 units

24. The function $f(x) = 3x^2 - 6x$ is concave upward in the interval

(a) $(-\infty, 1)$

(b) $(1, \infty)$

(c) $(0, 2)$

(d) $(-\infty, 0) \cup (2, \infty)$

25. $\int_0^{\pi} \cos^2 x \sin^3 x \, dx =$

(a) $\frac{2}{15}$

(b) $\frac{2\pi}{15}$

(c) $\frac{4}{15}$

(d) $\frac{4\pi}{15}$

26. $\int_0^{\infty} (1-x) e^{-x} dx =$

(a) e

(b) $\frac{1}{e}$

(c) 1

(d) 0

27. For the function $f(x, y) = e^{x^y}$,

$f_x(x, y) =$

(a) $(\log x) x^y e^{x^y}$

(b) $(\log y) x^y e^{x^y}$

(c) $y x^{y-1} e^{x^y}$

(d) $x^y e^{x^y}$

28. Let $x, y > 0$ and $\vec{F} = \frac{-y}{x^2 + y^2} \hat{i} + \frac{x}{x^2 + y^2} \hat{j}$.

Then $\nabla \times \vec{F} =$

(a) zero vector

(b) $\hat{i} + \hat{j}$

(c) $\hat{i} + \hat{j} + \hat{k}$

(d) $\hat{i} - \hat{j}$

29. The rank of an $m \times n$ matrix A has properties

(a) Rank $(A) \leq \min\{m, n\}$

(b) Rank $(A) \leq \max\{m, n\}$

(c) Rank $(A) \geq \min\{m, n\}$

(d) Rank $(A) \geq \max\{m, n\}$

30. If Rank $(A) < n$, then which one of the following is true?

(a) The reduced row echelon form of A has n non-zero rows

(b) The system $Ax = 0$ has some non-trivial solutions

(c) The matrix A has exactly n pivot positions

(d) None of the above

31. The system $5x + 7y = b_1$, $2x + 3y = b_2$ is consistent for

(a) all b_1 and b_2

(b) at least one b_1 and b_2

(c) no b_1 and b_2

(d) exactly one b_1 and b_2

32. If x and y are any vectors in Ker (A) and α is any scalar, then which one is/are also in Ker (A)?

(a) $x + y$ and αx

(b) Only $x + y$ but not αx

(c) Not $x + y$ but αx

(d) Neither $x + y$ nor αx

33. If A is invertible symmetric matrix, then

(a) A^{-1} is symmetric

(b) A^{-1} is skew-symmetric

(c) $A^2 - 2A + I = 0$

(d) None of the above

34. The polynomials $p = 1 - x$, $q = 5 + 3x - 2x^2$ and $r = 1 + 3x - x^2$

(a) form a linearly independent set in P_2

(b) form a basis set in P_2

(c) satisfy $3p - q + 2r = 0$

(d) form linearly dependent set in P_2

35. Which one of the following is subspace of $\mathbb{R}^3$?

(a) All vectors of the form $(a, 1, 1)$

(b) All vectors of the form $(a, b, 0)$

(c) All vectors of the form (a, b, c), where $b = a + c$

(d) All vectors of the form (a, b, c), where $b = a + c + 1$

36. For the group $\mathbb{Z}_{10}$ under addition modulo 10,

(a) $O(2) = 1$

(b) $O(2) = 5$

(c) $O(2) = 2$

(d) $O(2) = 3$

37. Which one of the following is correct?

(a) $U(10) \cong \mathbb{Z}_4$

(b) $U(5) \cong \mathbb{Z}_2$

(c) $U(6) \cong \mathbb{Z}_3$

(d) $U(7) \cong \mathbb{Z}_5$

38. Which one of the following is correct?

(a) The ring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is an integral domain

(b) The ring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is not an integral domain

(c) The ring $\mathbb{Z}_p$ of integer modulo a prime is not an integral domain

(d) The ring $M_2(\mathbb{Z})$ of 2×2 matrices over integer is an integral domain

39. What are the order and degree of the differential equation $(1 + y'^2)^{3/2} - ry'' = 0$?

(a) First order, second degree

(b) Second order, first degree

(c) Second order, second degree

(d) Third order, second degree

40. The solution of $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + \cos y + x} = 0$ is

(a) $y\sin x + (\sin y + y)x = c$

(b) $y\sin x + (\sin x + x) = c$

(c) $y = \sin x + y\sin y + c$

(d) None of the above

PART—B

41. The solution of $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial z}{\partial x} = \sin(4x+5)$ is

(a) $z = f_1(y+x) + f_2(y+4x) - \frac{1}{3}x\cos(4x+5)$

(b) $z = \frac{1}{3}x\cos(4x+5)$

(c) $z = f_1(y+x) + f_2(y+4x)$

(d) None of the above

42. The solution of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x + y$ is

(a) $z = \frac{2}{3}[(x+a)^{3/2} + (y-a)^{3/2}] + b$

(b) $z = \frac{2}{3}[(x+a)^{1/2} + (y-a)^{1/2}] + b$

(c) $z = [(x+a)^2 + (y-a)^2] + b$

(d) None of the above

43. The p-discriminant of $(x+c)^2 + y^2 = r^2$ is

(a) $y^2(y^2 - r^2) = 0$

(b) $r^2(y^2 - r^2) = 0$

(c) $y^2(x^2 - r^2) = 0$

(d) None of the above

44. The solution of the linear ordinary differential equation $y'' - 3y' + 2y = e^x$ is

(a) $a_1e^x + a_2e^{2x} - xe^x$

(b) $a_1e^x + a_2e^{-2x} - xe^x$

(c) $a_1e^{-x} + a_2e^{2x} - xe^x$

(d) $a_1e^x + a_2e^{2x} + xe^x$

45. The general solution of the differential equation $2yzdx + zxdy - xy(1+z)dz = 0$ is

(a) $x^2 y = cze^z$

(b) $x^2 y^2 = cze^z$

(c) $xy = ce^z$

(d) $x^2 y = cz$

46. The solution of the differential equation $(x+y)(dx - dy) = dx + dy$ is

(a) $x - y + c = \log(x+y)$

(b) $x + y + c = \sin(x+y)$

(c) $x - y + c = \cos(x+y)$

(d) $x - y + c = \exp(x+y)$

47. The solution of the initial value problem $\frac{dy}{dx} = e^{x+y}$, $y(1) = 1$ at $x = -1$ is

(a) 2

(b) 0

(c) -1

(d) 1

48. The number $\pi = 3 \cdot 14159265$ is approximated by $\frac{22}{7}$. The number of digits, this approximation is accurate, is

(a) 6

(b) 7

(c) 8

(d) 2

49. The minimum number of initial approximations required to implement Secant method to obtain the root of $f(x) = 0$ is

(a) 1

(b) 2

(c) 3

(d) 4

50. The necessary and sufficient condition for convergence to the iterative method $X^{(k+1)} = HX^{(k)} + c,\ k = 0, 1, 2, \cdots$ is

(a) $\rho(H) > 1$

(b) $\rho(H) < 1$

(c) $\rho(H) = 1$

(d) $\rho(H) = 0$

51. Gauss-Jacobi iterative method for the solution of the system of linear equation $AX = b$, converges for any initial vector $X^{(0)}$, provided

(a) A is Hermitian

(b) A is symmetric

(c) A is diagonally dominant

(d) A is singular

52. If two divisions of the interval [0, 1] is considered to evaluate the integral $\int_0^1 \frac{1}{3+2x}\,dx$, using Trapezoidal rule, then the actual error in the exact and approximate solution is

(a) 0·34560

(b) 0·00292

(c) 0·02230

(d) 0·29200

53. Which of the following is true in case of Simpson's one-third rule of integration?

(a) $\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

(b) $\int_a^b f(x)\,dx = \frac{h}{10}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

(c) $\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(a+b)\right]$

(d) $\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

54. The approximate numerical derivative of the function $f(x) = \sin(x)$ is given by the relation

(a) $10[\sin(x + 0{\cdot}1) - \sin(x)]$

(b) $20[\sin(x + 0{\cdot}1) - \sin(x)]$

(c) $\sin(x + 0{\cdot}1) - \sin(x)$

(d) $10[\sin(x - 0{\cdot}1) - \sin(x)]$

55. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. The probability that the equation will have real roots is

(a) $\frac{21}{216}$

(b) $\frac{43}{216}$

(c) $\frac{61}{216}$

(d) $\frac{63}{216}$

56. The sum of two non-negative quantities is equal to $2n$. The chance that their product is not less than $\frac{3}{4}$ times their greatest product is

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

57. The mean of a set of observations is 5. If each observation is multiplied by 3 and each product is decreased by 1, then the mean of new set of observations is

(a) $\frac{8}{3}$

(b) $\frac{2}{3}$

(c) 14

(d) 16

58. Let X be a random variable having Poisson distribution with mean λ. If its probability density function $P(X)$ follows the given condition $P(X=2)=9P(X=4)+90P(X=6)$, then the mean (λ) of the distribution is

(a) $\lambda = 1$

(b) $\lambda = -1$

(c) $\lambda = 2$

(d) $\lambda = 3$

59. If the two lines of regression, viz., y on x and x on y are $y=\frac{1}{3}x-11$, $x=dy+112$ respectively, then the limits within which the constant d must lie is given by

(a) $-1 \le d \le 1$

(b) $5 \le d \le 6$

(c) $0 \le d \le 3$

(d) $3 \le d \le 5$

60. A continuous random variable X has a probability density function

$$f(x)=\begin{cases} x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find a such that $P(X \le a) = P(X > a)$.

(a) $a=\left(\frac{1}{2}\right)^{\frac{1}{3}}$

(b) $a=\frac{1}{\sqrt{2}}$

(c) $a=\frac{1}{2}$

(d) None of the above

61. A random variable X has the following probability distribution values of X :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Then, $P(X \geq 6) =$

(a) $\frac{81}{100}$

(b) $\frac{19}{100}$

(c) $\frac{9}{100}$

(d) $\frac{91}{100}$

62. Consider the following LPP :

Minimize $Z = 2x_1 + x_2$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \leq 0$$

The optimal table of this LPP is

Basic	Z	x_1	x_2	s_2	R_1	R_2	S_3	Sol.
Z	1	0	0	$-1/5$	$(2/5)-M$	$(1/5)-M$	0	$12/5$
x_1	0	1	0	$1/5$	$3/5$	$-1/5$	0	$3/5$
x_2	0	0	1	$-3/5$	$-4/5$	$3/5$	0	$6/5$
s_3	0	0	0	1	1	-1	1	0

If we change the vector b from $(3, 6, 3)^T$ to $(5, 7, 3)^T$, then the new solution will be

(a) $x_1 = 8/5,\ x_2 = 1/5$

(b) $x_1 = 3/5,\ x_2 = 6/5$

(c) $x_1 = 3/5,\ x_2 = 1/5$

(d) $x_1 = 8/5,\ x_2 = 6/5$

63. Change in the cost coefficient c_j of an LPP will affect

(a) only objective function value

(b) only optimality condition

(c) optimality condition and objective function value

(d) optimality and feasibility condition both

64. If the primal LPP has an unbounded solution, then the dual problem

(a) has an optimal solution

(b) has no solution

(c) has an unbounded solution

(d) None of the above

65. Phase–1 of two-phase simplex method

(a) optimizes the objective function of a given problem

(b) gives a starting BFS

(c) is required if a variable is unrestricted in sign

(d) None of the above

66. If in a simplex table the relative cost $z_j - c_j$ is zero for a non-basic variable, then there exists an alternative optimal solution, provided

(a) it is the optimal simplex table

(b) it is a starting simplex table

(c) it can be any simplex table

(d) zero relative cost $z_j - c_j$ for a non-basic variable, does not imply an alternative optimal solution

67. In an LPP in standard form there are six variables and four constraints. Then the number of basic feasible solutions is

(a) 15

(b) ≤ 15

(c) ≥ 15

(d) Number of basic feasible solutions does not depend upon number of variables and number of constraints in the problem

68. For the problem :

Maximize $Z = 2x_1 + x_2 + 5x_3 + 6x_4$

subject to

$$2x_1 + x_2 + x_4 \leq 8$$
$$2x_1 + 2x_2 + x_3 + 2x_4 \leq 12$$
$$\text{all } x_i > 0$$

it is known that $x_1 = 0, x_2 = 0, x_3 = 4, x_4 = 4$ is the optimal solution. The optimal of the dual is

(a) $y_1 = 1, \ y_2 = 4$

(b) $y_1 = 4, \ y_2 = 4$

(c) $y_1 = 4, \ y_2 = 1$

(d) $y_1 = 0, \ y_2 = 0$

69. Let the primal be minimization. Let a feasible solution, which is not optimal, of primal has value 25. Then which of the following can be the value of dual at a feasible solution of dual?

(a) 24·5

(b) 25

(c) 26

(d) None of the above

70. The function $f(x) = \int_0^x \frac{t^2 - 6t + 8}{t^2 - 2t + 6} dt$ is increasing in the interval

(a) $(-\infty, 2) \cup (3, \infty)$

(b) $(2, 3)$

(c) $\left(-\infty, \frac{4}{3}\right) \cup (3, \infty)$

(d) $\left(\frac{4}{3}, 3\right)$

71. The series $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \cdots$

(a) converges to $\sqrt{2}$

(b) converges to 2

(c) converges to $2\sqrt{2}$

(d) diverges

72. Consider the functions

$$f(x) = \begin{cases} (1-e^{-x})^{-1}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Then at $x = 0$

(a) both f and g are continuous

(b) f is continuous but g is not

(c) g is continuous but f is not

(d) none of f and g is continuous

73. Define the sequence $\{f_n\}$ of functions, where $f_n(x) = 2^n x^n$. In order that the sequence $\{f_n\}$ is uniformly convergent in the interval $[0, k]$,

(a) $k < 1$

(b) $k < 1/2$

(c) $k < 2$

(d) $k < 4$

74. The value of $\int_0^\infty e^{-x^2} dx$ is

(a) $\sqrt{\pi/2}$

(b) $\pi/\sqrt{2}$

(c) $\sqrt{\pi}/2$

(d) $\pi/2$

75. At the point (0, 0), the function

$$f(x) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0 \quad , & (x, y) = (0, 0) \end{cases}$$

(a) is differentiable

(b) does not have partial derivatives

(c) is continuous but not differentiable

(d) is not continuous

76. If $\lim\limits_{n \to 0} a_n = 0$, then the series $\sum\limits_{n=1}^{\infty} a_n$

(a) converges but not absolutely

(b) may converge

(c) diverges

(d) converges absolutely

77. If f and g are real valued functions, then $\min(f, g)$ equals

(a) $\max\left(\frac{1}{f}, \frac{1}{g}\right)$

(b) $\frac{1}{\max(f, g)}$

(c) $\max(-f, -g)$

(d) $-\max(-f, -g)$

78. Consider $f(x) = \begin{cases} 0, & x < 0 \\ x^2/2, & 0 \le x \le 1 \\ 4x - 1, & x > 1 \end{cases}$

The function f is

(a) differentiable on $\mathbb{R}$

(b) not differentiable at 0 and 1

(c) not differentiable at 0

(d) not differentiable at 1

79. For $\phi(x, y, z) = 3x^2y - y^2z^2$, the value of $\nabla \cdot \phi$ at the point $(1, -2, 1)$ is

(a) $-12\hat{i} - 9\hat{j} - 16\hat{k}$

(b) $-12\hat{i} - 9\hat{j} + 16\hat{k}$

(c) $-12\hat{i} + 9\hat{j} - 16\hat{k}$

(d) $12\hat{i} - 9\hat{j} - 16\hat{k}$

80. Which one of the following is correct?

(a) Characteristic of $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is 0

(b) Characteristic of $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is 1

(c) Characteristic of $\mathbb{Z}_3[i] = a + bi,\ a, b \in \mathbb{Z}$ is 4

(d) Characteristic of $\mathbb{Z}[i] = a + bi,\ a, b \in \mathbb{Z}$ is 4

81. Let a mapping $\varphi : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{12}$ be defined as $\varphi(x) = 3x$, then

(a) $\text{Ker}(\varphi) = \{0\}$

(b) $\text{Ker}(\varphi) = \{0, 2\}$

(c) $\text{Ker}(\varphi) = \{0, 4\}$

(d) $\text{Ker}(\varphi) = \{0, 4, 8\}$

82. Let $p_1 = t + 1$, $p_2 = t - 1$, $p_3 = t^2 - 2t + 1$ and $v = 2t^2 - 5t + 9$, then

(a) $v = 3p_1 - 4p_2 + 3p_3$

(b) $v = 3p_1 - 4p_2 + 2p_3$

(c) $v = 3p_1 + 4p_2 + 3p_3$

(d) $v = 3p_1 + 4p_2 - 3p_3$

83. Let $f : R \rightarrow R$ defined by $f(x) = 2x - 3$, then

(a) $f^{-1}(x) = (x + 3)/2$

(b) $f^{-1}(x) = (x - 3)/2$

(c) $f^{-1}(x) = (x + 2)/3$

(d) $f^{-1}(x) = (x - 2)/3$

84. Let U and V are distinct four-dimensional subspaces of a vector space W with $\dim(W) = 6$, then

(a) $\dim(U \cap V) = 3$ or 2

(b) $\dim(U \cap V) = 3$ or 4

(c) $\dim(U \cap V) = 3$ or 5

(d) $\dim(U \cap V) = 3$ or 6

85. Which one of the following is correct?

(a) $\mathbb{Z}$ is not a subring of Q

(b) Q is not a subring of $\mathbb{R}$

(c) $\mathbb{Z}/n\mathbb{Z}$ is a subring of $\mathbb{Z}$

(d) $2\mathbb{Z}$ is a subring of $\mathbb{Z}$

86. Which one of the following is correct?

(a) The ring $\mathbb{Z}$ has a unique zero divisor

(b) Only units of ring $\mathbb{Z}$ are $\{1, -1\}$

(c) $\mathbb{Z}/n\mathbb{Z}$ can never be a field

(d) $\mathbb{Z}/n\mathbb{Z}$ has no zero divisor

87. S_3, the symmetric group of degree n has

(a) two Sylow 2-subgroup

(b) three Sylow 2-subgroup

(c) four Sylow 2-subgroup

(d) no Sylow 2-subgroup

88. Let G be a set of positive real numbers with binary operation $*$ defined by $a * b = ab / 7$ for all $a, b \in G$, then the identity element of G is

(a) $1/a$

(b) 2

(c) $2/a$

(d) 7

89. In $\mathbb{Z}/n\mathbb{Z}[x]$, the polynomial

(a) $x^2 + x + 1$ is reducible

(b) $x^3 + x + 1$ is reducible

(c) $x^2 + 1$ is reducible

(d) $x^2 + 1$ is irreducible

90. The generator of group $\mathbb{Z}/48\mathbb{Z}$ is

(a) 2

(b) 3

(c) 4

(d) 7

91. The order of $\overline{30}$ in $\mathbb{Z}/54\mathbb{Z}$ is

(a) 6

(b) 7

(c) 8

(d) 9

92. Which of the following pairs in symmetric group are not conjugate?

(a) $\sigma_1 = (1\ 2)(3\ 4\ 5)$ and $\sigma_2 = (1\ 2\ 3)(4\ 5)$

(b) $\sigma_1 = (1\ 5)(3\ 7\ 2)(10\ 6\ 8\ 11)$ and $\sigma_2 = (3\ 7\ 5\ 10)(4\ 9)(13\ 11\ 2)$

(c) $\sigma_1 = (1\ 5)(3\ 7\ 2)(10\ 6\ 8\ 11)$ and $\sigma_2 = \sigma_1^3$

(d) $\sigma_1 = (1\ 3)(2\ 4\ 6)$ and $\sigma_2 = (3\ 5)(2\ 4)(5\ 6)$

93. The following differential equation

$$\frac{d^2y}{dx^2} + \sin(x+y) = \sin(x)$$

is

(a) non-linear and non-homogeneous

(b) linear and non-homogeneous

(c) non-linear and homogeneous

(d) None of the above

94. The solution of the differential equation $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(a) $x^3y^2 + \frac{x^2}{y} = \frac{x}{y}$

(b) $x^3y^2 + \frac{x^2}{y^2} = x$

(c) $x^3y^2 + \frac{x^2}{y} = \text{constant}$

(d) None of the above

95. The Rodrigues' formula for Legendre polynomial is

(a) $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$

(b) $P_n(x) = \frac{1}{n!2}\frac{d^n}{dx^n}(x^2 - 1)^n$

(c) $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2 - 1)^{n-1}$

(d) $P_n(x) = \frac{d^n}{dx^n}(x^2 - 1)^n$

96. The orthogonal trajectory of the hyperbola $x^2 - y^2 = c$ is

(a) $xy = c$

(b) $xy^2 = c$

(c) $x^2y = c$

(d) None of the above

97. The relation $z = (x+a)(y+b)$ represents the partial differential equation

(a) $z = \frac{p}{q}$

(b) $z = pq$

(c) $z = p - q$

(d) None of the above

98. The partial differential equation $U_t = U_{xx}$ is

(a) one-dimensional heat conduction equation

(b) Laplace equation

(c) wave equation

(d) two-dimensional heat conduction equation

99. The general solution of the differential equation, $2yzp + zxq = 3xy$ is

(a) $\varphi(x^2 - 2y^2,\ 3y^2 - z^2) = 0$

(b) $\varphi(x^2 - 2y^2,\ 3y^2 - z) = 0$

(c) $\varphi(x - 2y^2,\ 3y - z) = 0$

(d) $\varphi(x^2 - 2y,\ 3y^2 - z^2) = 0$

100. In the region $x > 0$, $y > 0$, the partial differential equation

$$(x^2 - y^2)\frac{\partial^2 u}{\partial x^2} + 2(x^2 + y^2)\frac{\partial^2 u}{\partial x \partial y} + (x^2 - y^2)\frac{\partial^2 u}{\partial y^2} = 0$$

is

(a) elliptic

(b) parabolic

(c) hyperbolic

(d) None of the above

★★★

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